07 POLYNOMIALS

1. ALGEBRAIC LANGUAGE OR SYMBOLIC LANGUAGE

This is the language that expresses numerical relations in which there are variable quantities or unknowns. Because they don't have a fixed value; or because it is not known we use letters to name these.

Formulas and equations are the most important examples.

An algebraic expression is a combination of letters, numbers and parentheses, related to operations. The elements of an algebraic expression are:

- Terms: each of the summands.
- Constant term: Only has a numerical part.
- Variables: the unknown quantities.
- Coefficients: the multiplying number of each variable.

MONOMIALS

The elemental algebraic expression consisting of only one term.

Example: $\frac{2}{3}x^5$

2/3 is the coefficient, x^5 is the literal part and 5 is the degree.

Similar monomials are those that have the same literal part.

POLYNOMIALS

These are the sum of monomials.

The polynomial degree is the highest degree that it contains.

Binomial is a two-term polynomial, trinomial is a three-term polynomial and so on.

NUMERICAL VALUE OF A POLYNOMIAL

This is the value obtained by replacing the variable with a number and performing the operations.

2. OPERATIONS WITH MONOMIALS

ADDING AND SUBTRACTING MONOMIALS

If the monomials are similar we can add or subtract the coefficients and leave the common literal part.

If the monomials are not similar we get a polynomial.

The opposite of a monomial is the same monomial with the opposite sign.

PRODUCT

We multiply the coefficients and elsewhere the variables. The result is a new monomial whose degree is the sum of the factor degrees.

DIVISION

We divide the coefficients and divide the variables. The result is a new monomial whose degree is the subtraction of the degrees.

POWER

To raise a monomial to a power we raise the coefficient to the power and we multiply the exponent of the variable by the exponent of the power.

PRODUCT BY A POLYNOMIAL. DISTRIBUTIVE AND COMMON FACTOR

To multiply a monomial by a polynomial we multiply the monomial by each term according to the distributive property.

In the opposite way we can extract a common factor of a polynomial.

3. OPERATIONS WITH POLYNOMIALS

ADDITION

We add the terms with the same degree.

The opposite of a polynomial is the polynomial with opposite coefficients.

SUBTRACTION

We add the opposite because subtracting is the same as adding the opposite.

MULTIPLICATION

We multiply every term of a polynomial by every term of the other. Then we simplify like terms.

EXERCISES AND PROBLEMS

1. ALGEBRAIC LANGUAGE OR SYMBOLIC LANGUAGE

2. OPERATIONS WITH MONOMIALS

3. OPERATIONS WITH POLYNOMIALS

1. Given the polynomials: $P(x) = x^4 - 2x^2 - 6x - 1$; $Q(x) = x^3 - 6x^2 + 4$ and $R(x) = 2x^2 - 2x - 2$

a. Calculate: $P(x) + 2\,Q(x) - R(x)$

b. Multiply: $Q(x) \cdot R(x)$

2. Given the polynomials: $P(x) = x^4 - 2x^2 - 6x - 1$; $Q(x) = x^3 - 6x^2 + 4$ and $R(x) = 2x^2 - 2x - 2$

a. Calculate: $3 \cdot P(x) - R(x)$

b. Multiply: $P(x) \cdot R(x)$

3. Given the polynomials: $P(x) = x^4 - 2x^2 - 6x - 1$; $Q(x) = x^3 - 6x^2 + 4$ and $R(x) = 2x^2 - 2x - 2$

a. Calculate: $P(x) - 2\,R(x)$

b. Multiply: $P(x) \cdot Q(x)$

4. Find the numerical value for $a = 1$; $b = 0$ and $x = 2$ for the following expressions:

a) $a + b^2 - x =$

b) $x^2 + 3x - 1 =$

c) $ax + a =$

d) $b^2 - 7a =$

5. Given the polynomials: $P(x) = x^4 - 2x^2 - 6x - 1$; $Q(x) = x^3 - 6x^2 + 4$; $R(x) = 2x^2 - 2x - 2$

Calculate:

a) $P(x) + Q(x) - R(x) =$

b) $P(x) + 2\,Q(x) - R(x) =$

c) $Q(x) + R(x) - P(x)=$

6. Multiply:

a) $(x^4 - 2x^2 + 2) \cdot (x^2 - 2x + 3)$

b) $(3x^2 - 5x) \cdot (2x^3 + 4x^2 - x + 2)$

c) $(2x^2 - 5x + 6) \cdot (3x^4 - 5x^3 - 6x^2 + 4x - 3) =$